sponds to the case where all the s particles reach the surface without collision, and for Kn $\rightarrow$ 0 the result is close to $c_x = 0.88$, the value given by the modified Newtonian theory for $M \rightarrow \infty$.

Qualitatively flow of elastic particles over a sphere is close to flow of a rarefied gas over a sphere, and with the model adopted we can compute the flow for Kn corresponding to the transition regime. This gives a basis to expect that the three-component model presented can be used to compute approximately the flow of a rarefied gas over bodies in the transition regime. To do this we need to use more realistic laws for the interaction of particles with the surface and to compute the viscosity of the t component. In addition, in order to be able to vary the Mach number we must introduce random motion of particles in the unperturbed flow, since in the model presented the flow of s particles corresponds to the limiting hypersonic case of $M \rightarrow \infty$.

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NONSELF-SIMILAR JET OF A NON-NEWTONIAN LIQUID

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The results of an analysis of the propagation of a two-dimensional submerged jet of a non-Newtonian liquid over the entire zone of its development are given within the framework of the boundary theory.

Jet flow is encountered in many technological applications. The pressing problem of analyzing non-Newtonian jet flow is created, in particular, by the broadening of the scope of application of polymers. Moreover, one must not forget the analogy between a turbulent flow and a non-Newtonian liquid with changes in the integral hydrodynamic parameters.

A self-similar solution was obtained earlier for a two-dimensional jet of a non-Newtonian liquid [1]. We shall investigate here the development of a two-dimensional jet of a non-Newtonian liquid throughout the entire region of its propagation by means of numerical calculations, using the method of local similarity. The Ostwald—de Ville model is used for approximating the flow rheology. Practical application of this model is justified in many cases of actual flow, for instance, polymer flow.

The initial equations of momentum transport and continuity of the submerged two-dimensional jet of a non-Newtonian liquid are given by

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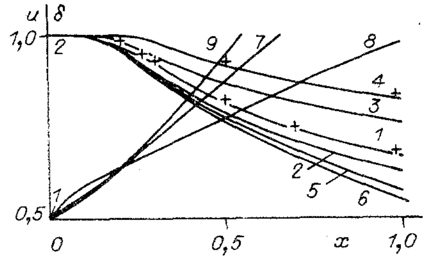

Fig. 1

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = m \frac{\partial}{\partial y} \left(\left| \frac{\partial u}{\partial y} \right|^{N-1} \frac{\partial u}{\partial y} \right), \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

The boundary conditions for the jet flow are $\partial u/\partial y = 0$, $v = 0$, $y = 0$, $x > 0$, $u \to 0$, $y \to \infty$. The initial conditions are $u = 1$, $x = 0$, $|y| < d/2$, $u = 0$, $|y| \geq d/2$.

The presence of the integral invariant (jet momentum) facilitates a substitution of variables convenient for numerical calculations [2],

$$\xi = x, \quad \eta = \int_0^y u^2 \, dy.$$

In terms of the new variables, the nonself-similar problem of an immersed, two-dimensional jet of a non-Newtonian liquid is stated as follows:

$$\begin{gathered} \frac{\partial u}{\partial \xi} = \left[N u \frac{\partial^2 u}{\partial \eta^2} + (2N-1) \left(\frac{\partial u}{\partial \eta} \right)^2 \right] \left| \frac{\partial u}{\partial y} \right|^{N-1} u^{2N}, \\ v = -Nu \int_0^\eta \frac{\partial}{\partial \eta_1} \left[u^2 \frac{\partial u}{\partial \eta_1} \right] \left| \frac{\partial u}{\partial \eta_1} \right|^{N-1} u^{2N-4} \, d\eta_1, \quad y = \int_0^\eta \frac{d\eta}{u^2}, \\ u = 1, \ \xi = 0, \ 0 \leqslant \eta < 1, \ u = 0, \ \xi = 0, \ \eta = 1, \ \partial u/\partial \eta = v = 0, \ \eta = 0, \\ \xi \geqslant 0, \ u = 0, \ \eta = 1. \end{gathered} \tag{1}$$

Problem (1) was solved numerically by using the method of straight lines. The integration band was subdivided into P bands (P = 20). One should mention the advantage of the substitution of variables — the conversion of an infinite integration region into a finite band — which makes it possible to avoid the buildup of numerical errors. The direct integration was performed by using the Runge–Kutta method for a system of P − 1 ordinary differential equations. The characteristic results of the numerical solution are given in Figs. 1-3.

Figure 1 shows the variation of the axial velocity and thickness of the jet along its axis for different values of the index of the non-Newtonian character N: 1)-9) N = 1; 0.75; 1.5; 2; 0.5; 0.25; 1; 2; 0.25, respectively; 7)-9) δ. The changes in the axial velocity as the liquid deviates from the Newtonian behavior consist in the following. For dilatant liquids ($N > 1$), the axial velocity diminishes to a lesser extent in comparison with a Newtonian liquid. In the limiting case ($N \to \infty$), the axial velocity does not change along the x axis, and we have jet-stream core flow conditions. For large N values, the jet core spreads more slowly than in the case of smaller N values. For a pseudoplastic liquid ($0 < N < 1$), the axial velocity diminishes more quickly than for a Newtonian liquid; there is an initial region $0 \lesssim x \lesssim 0.2$ where the jet core of a pseudoplastic liquid differs little from a Newtonian core. Changes in the jet thickness of a pseudoplastic liquid occur more quickly, and those of a dilatant liquid more slowly, than in the case of a Newtonian liquid beyond the inversion point $x_{inf} \sim 0.2$; for $0 < x < x_{inf}$, the opposite occurs.

Figure 2 shows the jet velocity profiles for different values of N: 1)-4) N = 2; 1.5; 1; 0.25 at the distance x = 0.3 from the initial cross section of the jet. We readily see that there is little difference between the profiles in a wide range of N values (0.25-2) of the rheological model, which is entirely suitable for practical applications. In other words, the numerical solution serves as a basis for using the profile of a Newtonian liquid to estimate the constants of a self-similar non-Newtonian jet.

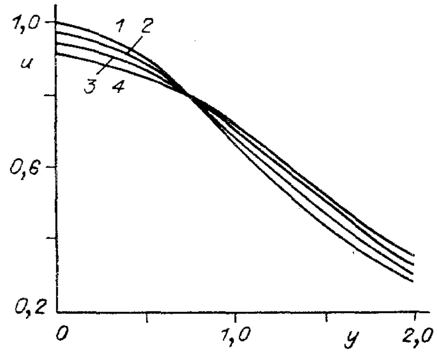

Fig. 2

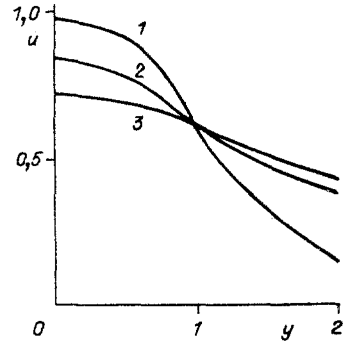

Fig. 3

Figure 3 shows the behavior of the velocity profiles of a pseudoplastic liquid with increasing distance from the initial cross section: 1)-3) x = 0.2; 0.4; 0.6; N = 0.25. The spreading of the jet increases with distance from the initial cross section; much of it occurs in the region around the inversion point x_{inf}.

Analysis of the more distant region of jet propagation is based on the local similarity method [3],

$$u = u_m(x)f'(\varphi),\ \varphi = y/\delta(x).$$

In terms of the similarity variable, the axial velocity is $u_m = Bx^{-1/3N}$, while the jet thickness is

$$\delta = Cx^{2/3N}\left(B = \alpha^{-(N+1)/3N}, C = \alpha^{(2-N)/3N},\ \alpha = \frac{6N}{\gamma_1{}^n},\ \gamma_1 = \int\limits_{-\infty}^{\infty} f'^2\, d\varphi = \frac{4}{3}\right).$$

For an analytical estimate of the near region of jet development, we use the concept of the polar distance γ, which can be assigned by using the mass discharge at the initial cross section of the jet [4]:

$$\gamma = M_0^3 d^{N-1}/(36mu_0^{N-1}I).$$

Introducing the abscissa with a shift, we obtain the expressions for the axial velocity and the jet thickness that are in satisfactory agreement with the numerical solution (see Fig. 1, where the crosses pertain to the analytical solution):

$$u_m = Bx^{-1/3N}(1+\gamma/x)^{-1/3N},\ \alpha = 9/2n,\ \delta = Cx^{2/3N}(1+\gamma/x)^{2/3N},$$
$$\gamma = \gamma_0/N^{2,2},\ \gamma_0 = 0.05,\ n = n_0 e^{1/N},\ n_0 = 8.3.$$

In conclusion, it should be noted that the above analysis of the propagation of a two-dimensional nonself-similar jet of a non-Newtonian liquid in submerged space is also of practical, besides theoretical, interest. In addition to the above applications, one could also mention the use of magnetic liquids, where the rheological effect plays a substantial role, while the small scale in these applications require that the flow parameters be known for the entire flow development zone.

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